Copyright © 2014 The Regents of the University of California

The research reported here was supported by the Institute of Education Sciences, U.S. Department of Education, through Grant R305C080015 to the National Center for Research on Evaluation, Standards, and Student Testing (CRESST). The opinions expressed are those of the authors and do not represent views of the Institute or the U.S. Department of Education.
Overview of the Notebook

The goal of this notebook is to provide you with the necessary information to use this game with your students. In this notebook, you will find:

1) **Overview of the Game**: This section provides a brief introduction about the game including how to play and the math topics covered by the game.

2) **The Screen**: This page of the notebook shows an example of a typical screen and points out elements of the game that will help your students play such as game resources and screen features that may be important to know about as your students play the game.

3) **Suggested Topics for Classroom Instruction**: This list provides an overview of the math topics related to this game. Depending on your purpose for using the game (e.g., review, introduction, etc.), the math topics can be used as preview topics to the game or something to go over with students after they have played the game. You might teach the lesson to the whole class, a small group, or an individual student who is having difficulty.

4) **Common Errors Students Might Make in the Game**: This is a list of the most common errors students might make in this game. Review this list and watch carefully for these errors in the game. In many cases, the mistakes students make in the game provide hints to you about their deeper understanding and misconceptions about fractions.

5) **Sample Lesson Plan**: This section provides a sample lesson that could be used as a guide for classroom instruction.

6) **FAQ**: This section provides you with frequently asked questions about this game.

7) **Standards Assessed**: This list provides an overview of the Common Core State Standards for Mathematics addressed in this game. Both middle school and elementary school standards may be listed.
Overview of the Game

Topics covered in the game: conceptual understanding of multiplying and dividing by fractions and whole numbers.

The game *Tlaloc’s Book* challenges players to overcome barriers they encounter during their adventurous journey through a mysterious jungle. Players move through the jungle by choosing to move left, right, or jump. Players encounter barriers which prevent them from moving forward or reaching valuable items required to overcome the barriers. By using the numbers and operations they collect and store in their backpack, players can change the height of platforms. The player uses the platforms to overcome barriers.

All tasks in *Tlaloc’s Book* involve using a multiplicative comparison to change the platform height to a target height. The platform height is denoted on the platform, and the target height is denoted on the signpost. The height of the base of the signpost is to scale with the height denoted on the platform. The height on the platform is updated with each player action, which is composed of the choice of an available operator, either multiplication or division, and the choice of a scalar from the backpack.

Each action players perform is recorded as math notations on the scroll and the players can view this information whenever they want to. For example, they can track their action history and undo their actions by choosing the appropriate operation and scalar. When the players solve the problem correctly, the pertinent math equations appear on the signpost. The signpost flashes for a second to attract the player’s attention and the numbers used to solve the problem are stored in the treasure box. The numbers used to solve the problem go into the treasure box so that if there is another platform task on the same level, the player must solve it using a different strategy than was used on the first platform.

Levels 1-11: Multiplication and division with whole numbers
Levels 12-28: Multiplication and division with fractions
Levels 29-45: Multiplication and division with unit and non-unit fractions
The Screen

The current level you are on and the number of treasure chests you have collected will show up here.

The resources you collect will show up here as you collect them.

Click here to turn the sound on or off.

Instructions about how to play and what to do will show up in this box.

This tells you how high the pillar needs to be. This is your goal.

These are resources you need to collect. Jump and hit them to collect them.

Click here to go to the main menu to select a level or to see how many treasures you have collected.

Click here to restart the level.

Clicking on this scroll will show you what operations you've carried out.

This is the pillar that you must make taller or shorter by collecting resources. Click on this to carry out an operation.
Suggested Topics for Classroom Instruction

Below are the topics associated with this game. If students are having a particular type of trouble with a game, you might want to address that topic with the whole class or individual students.

Relationships Between Fractions and Whole Numbers
The game begins with whole numbers and moves to the use of fractions when a fraction results from the division of two whole numbers. So, fractions result from divisions and can describe quantities just as whole numbers do. Further, fractions can be used as scalars, just as whole numbers can be.

Inverse Operations
The game uses known multiplication and division facts to emphasize the inverse relationship of the operations. For instance, $24 \div 8 = 3$ because $3 \times 8 = 24$. The two statements are inextricably related, and this relationship is maintained after fractions are introduced.

Opposite Effects of Inverse Operations
Multiplying by 8 can be “undone” by dividing by 8. Multiplying by 8 makes a pillar 8 times taller in the game, and dividing by 8 makes it 8 times shorter (or more familiarly “1/8 as tall”).

Unit Fractions
A unit fraction is a fraction with a numerator of 1. The unit fraction $1/5$ tells us that a unit or whole has been divided into 5 equal pieces, each piece being $1/5$ of the whole unit.

Reciprocals
Multiplication and division have the same effect if we use reciprocals. The product of two reciprocals is 1. Multiplying by any rational number has the same effect as dividing by its reciprocal. So, multiplying by $1/3$ has the same effect as dividing by 3. This can be extended to non-unit fractions, as well. Multiplying by $2/5$ has the same effect as dividing by $5/2$. 
Scaling
To make a quantity larger or smaller, we must consider both the scaling operation and the scale factor. In this game, multiplying a positive integer by a number greater than 1 increases the quantity, and multiplying a positive integer by a number less than 1 decreases the quantity. Dividing a positive integer by a number greater than 1 decreases the quantity, and dividing a positive integer by a number less than 1 increases the quantity.
Common Errors
Students Might Make In the Game

The pillar is the right size, but the avatar can’t get across
Have the student move the avatar away from edge of the cliff in order to do a “running jump.”

Incorrect operator used
Students who use an incorrect operator can be directed to look at the scroll to see what operations were performed to understand the effect of the operators on the pillars.

Numbers used in the incorrect order
(For example, to get 5 from 8, you must divide 8 by 8 and then multiply by 5. To switch the order would arrive at the wrong number.)

Pillar height
Students may try to make a pillar shorter (or taller) using division (or multiplication) based on an overgeneralization of the effects of these operations with whole numbers. Students may not notice that they must make the pillar the exact height of the target (which is written on the signpost).

Forgetting to click on the pillar
Sometimes students forget to click on the pillar when they are trying to multiply or divide. If they do not click on the pillar, the operation will not be complete.

Constantly clicking on the same pillar
A pillar gets taller (or shorter) with every consecutive click after the initial click (for example, when clicking on 3, x, and the pillar 1/3, the pillar will grow taller to 1 from 1/3. Another click on the same pillar after that will make it grow to 3 because the calculation is still set at 3 multiplied by the value of the current pillar which is 1, in which case the pillar height is tripled). Direct students to look at the unit value on the pillar as that value changes with every click.

Division is not commutative
The number on the pillar is always set to be the dividend or the number that is being divided. The number chosen from the top is always set to be the divisor. For example, a student may expect that by clicking on 6, ÷, and the pillar 3, the result will be 2 because 6/3= 2 but because the number on the pillar is set to always be the divisor, it will actually be 3/6 = 1/2.
Sample Lesson Plan

When using Tlaloc’s Book with your students, the focus of your lesson could vary depending on if you wanted to use the game for review or initial instruction. Below is a sample lesson plan intended for students who have previously learned about fractions and are now using Tlaloc’s Book as practice of the concepts.

The lessons below can be done on consecutive days or spaced further apart as necessary. Lessons can also be combined depending on how fast students play each level and/or how proficient students are with each topic.

Lesson 1: Quick review of one or more of the following: the relationship between fractions and whole numbers, inverse operations, opposite effects of inverse operations, unit fractions, reciprocals, and scaling (list from Math Topics section). Next, introduce the game to students at front of the class (play first few levels together).

Lesson 2: Quick review of multiplication and division with whole numbers and fractions. Students play levels 1-28 on individual computers. At the end of the class, debrief student difficulties and successes.

Lesson 3: Quick review of student difficulties and successes from the day before; have students play levels 29-45. After students have completed all levels, review the topics of the game and make explicit the connection between the mathematics from the game and the mathematics learned in the classroom.
Tlaloc’s Book FAQs

Can students skip levels?
No. Students are only allowed to play the level after the last completed level. However, students can return to previously completed levels if they want to replay a level.

Can I help students while they play the game?
Yes.

We have found that students seem to learn best when they make sense of the math in the game on their own in order to succeed at each level.

That said, you are probably in the best position to determine how to help. If students are getting frustrated or unmotivated to play, it may be best to help them through one or more levels until they better understand the goal, mechanics, and math of the game.
Standards Addressed in the Game

Common Core State Standards
Mathematics (2010)
Grade 3
Operations and Algebraic Thinking
3.OA.B: Understand properties of multiplication and the relationship between multiplication and division.
3.OA.B.6: Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

Number and Operations—Fractions
3.NF.A: Develop understanding of fractions as numbers.
3.NF.A.3c: Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.

Grade 5
Number and Operations—Fractions
5.NF.B: Apply and extend previous understandings of multiplication and division.
5.NF.B.4: Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
5.NF.B.4a: Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)
5.NF.B.5: Interpret multiplication as scaling (resizing), by:
5.NF.B.5b: Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n × a)/(n × b) to the effect of multiplying a/b by 1.
5.NF.B.7: Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
5.NF.B.7a: Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the
quotient. Use the relationship between multiplication and division to explain that \( \frac{1}{3} \div 4 = \frac{1}{12} \) because \( \frac{1}{12} \times 4 = \frac{1}{3} \).

5.NF.B.7b: Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for \( 4 \div \frac{1}{5} \), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \( 4 \div \frac{1}{5} = 20 \) because \( 20 \times \frac{1}{5} = 4 \).