Maximum Marginal Likelihood Bifactor Analysis with Estimation of the General Dimension as an Empirical Histogram

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Outline

Introduction
Empirical histogram in item bifactor analysis
Simulation study
Empirical example
Conclusion
Woods and Thissen (2006) discussed a number of scenarios where the ubiquitous latent variable normality assumption may be violated in IRT applications.

For instance, the population may be heterogeneous, or the distribution of the target construct may have a theoretical distribution that is not bell-shaped, e.g., depression in the general population.

Woods in a series of papers in the 2000’s proposed solutions that explicitly modeled latent variable non-normality, including the use of empirical histogram, Ramsay-Curve IRT, and Davidian-Curve IRT.

Discussions have been restricted to unidimensional IRT.
A Passage-Based Reading Assessment
NIDA Smoking Module for PROMIS®

Project goal: to develop, evaluate, and standardize item banks to assess cigarette smoking behavior and constructs associated with smoking for both daily and non-daily smokers

The bifactor structure for the Dependence/Craving domain

- G: dependence/craving (55)
- S1: first cig of the day (3)
- S2: automatic/mindless (4)
- S3: heavy (5)
- S4: out of control (8)
- S5: withdrawal (8)
- S6: cravings (3)
- S7: if I couldn’t smoke (4)
- S8: can’t quit (2)
- S9: temptations (5)
- S10: consistency (6)
- S11: when not allowed (3)
Dichotomously Scored Responses

Let $\theta$ be the general dimension and $\xi_s$ be the $s$th specific dimension, $s = 1, \ldots, S$.

Take the 3PL (a misnomer really) model as an example:

$$T_i(1|\theta, \xi_s) = g_i + \frac{1 - g_i}{1 + \exp[-(c_i + a_i^0 \theta + a_i^s \xi_s)]}.$$  

2PL is a special case:

$$T_i(1|\theta, \xi_s) = \frac{1}{1 + \exp[-(c_i + a_i^0 \theta + a_i^s \xi_s)]}.$$  

1PL is a further special case (note parameter constraint):

$$T_i(1|\theta, \xi_s) = \frac{1}{1 + \exp[-(c_i + a_i^0 \theta + a_i^s \xi_s)]}.$$
Graded Responses

For an item with \( K_i \) ordered polytomous responses.

Let the response categories be coded as \( k = 0, \ldots, K_i - 1 \). The cumulative response probability for item \( i \) in categories \( k \) and above is:

\[
T_i^+(k|\theta, \xi_s) = \frac{1}{1 + \exp[-(c_{ik} + a_i^0 \theta + a_i^s \xi_s)]},
\]

for \( k = 1, \ldots, K_i - 1 \).

Define the boundary cases \( T_i^+(0|\theta, \xi_s) = 1 \) and \( T_i^+(K|\theta, \xi_s) = 0 \), the category response probabilities are:

\[
T_i(k|\theta, \xi_s) = T_i^+(k|\theta, \xi_s) - T_i^+(k + 1|\theta, \xi_s),
\]

for \( k = 0, \ldots, K_i - 1 \).
Conditional Probabilities

Let $U_i$ be a random variable whose realization $u_i$ is a response to item $i = 1, \ldots, I$. The probability mass function of $U_i$, conditional on the latent variables, is multinomial:

$$P(U_i = u_i | \theta, \xi_s) = \prod_{k=0}^{K_i-1} [T_i(u_i | \theta, \xi_s)]^{1_k(u_i)},$$

where $1_k(u_i)$ is an indicator function such that

$$1_k(u_i) = \begin{cases} 1, & \text{if } k = u_i \\ 0, & \text{otherwise} \end{cases}.$$

Under the conditional independence assumption:

$$P\left(\bigcap_{i=1}^{I} U_i = u_i \bigg| \theta, \xi_s\right) = \prod_{i=1}^{I} P(U_i = u_i | \theta, \xi_s).$$
Dimension Reduction

Assuming that $h(\theta)$ is the distribution of the primary latent variable, and $g(\xi_s)$ the distribution of latent variable $\xi_s$, the marginal response pattern probability is:

$$P\left(\bigcap_{i=1}^{I} U_i = u_i\right)$$

$$= \int \prod_{s=1}^{S} \int \prod_{i \in \mathcal{H}_s} P(U_i = u_i | \theta, \xi_s) g(\xi_s) \, d\xi_s \, h(\theta) \, d\theta$$

(as per Gibbons & Hedeker, 1992), and the $S + 1$ dimensional integral can be transformed to a constant multiple of 2-dimensional integrals, where $\mathcal{H}_s$ collects together all items that load on specific dimension $s$. 

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If we let $h(\theta)$ be characterized by an empirical histogram (Bock & Aitkin, 1981), and leave all the $g(\xi_s)$ as univariate normal...

There are enough items loading on the general dimension, permitting an accurate characterization of its shape. The specific dimensions still account for residual dependence above and beyond the general dimension. With more than one groups, the latent variables’ location and scale may also be estimated, permitting scaling and DIF analyses.
Simulation Studies

The generating model either had normal latent variables, or a severely nonnormal (bimodal) general dimension, chosen to match real assessment data obtained from a substance abuse treatment outcomes project.

$N = 2000$ item responses were generated from the graded response model with 5 categories.

The total number of items was either 15 or 30.

For 15 items, there were 6 specific dimension.

For 30 items, there were 12 specific dimensions.

In each condition, 500 replications were attempted.

2 bifactor models were fitted: one assuming normality, and one based on empirical histogram estimation.
Latent Density Recovery

Non-normal Generating Model

Normal Generating Model
Latent Density Recovery

Simulation

Non-normal, 15-item

Non-normal, 30-item

Normal, 15-item

Normal, 30-item
23 items in total

5 specific dimensions:
- I feel like part of a group when I'm around other smokers.
- Smoking makes me feel more self-confident with others.
- I enjoy the social aspect of smoking with other smokers.
- When I see other people smoking I want a cigarette.
- Quit smoking I will be less welcome around my friends.
- Of the people I care about…(none, some, most, all) are smokers

1 overall dimension on which all 23 items load

2 groups:
- Daily smokers ($N = 3201$)
- Non-daily smokers ($N=1183$)
Social Factors 2-Group EH Bifactor Model

Empirical Data

PROMIS® Smoking (Social Factors Domain)
Summary

Item bifactor model can efficiently account for residual dependence without dramatically increasing the computational burden.

Empirical histogram can be effectively combined with item bifactor analysis.

Simulations suggest that one may as well always estimate bifactor models with the general dimension characterized as an empirical histogram for reduced bias in item parameter estimates.

The models and estimation algorithms can be extended to more than one groups and are already available in flexMIRT™ (www.flexmirt.com).
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